Radiation Sources

▪ Problem 1.1. Radiation Energy Spectra: Line vs. Continuous

Line (or discrete energy): a, c, d, e, f, and i.
Continuous energy: b, g, and h.

▪ Problem 1.2. Conversion electron energies compared.

Since the electrons in outer shells are bound less tightly than those in closer shells, conversion electrons from outer shells will have greater emerging energies. Thus, the M shell electron will emerge with greater energy than a K or L shell electron.

▪ Problem 1.3. Nuclear decay and predicted energies.

We write the conservation of energy and momentum equations and solve them for the energy of the alpha particle. Momentum is given the symbol "p", and energy is "E". For the subscripts, "al" stands for alpha, while "b" denotes the daughter nucleus.

$$p_{al} + p_b = 0 \qquad \frac{p_{al}^2}{2\,m_{al}} = E_{al} \qquad \frac{p_b^2}{2\,m_b} = E_b \qquad E_{al} + E_b = Q \quad \text{and} \quad Q = 5.5\ \text{MeV}$$

Solving our system of equations for E_{al}, E_b, p_{al}, p_b, we get the solutions shown below. Note that we have two possible sets of solutions (this does not effect the final result).

$$E_b = 5.5\left(1 - \frac{m_{al}}{m_{al} + m_b}\right) \qquad E_{al} = \frac{5.5\,m_{al}}{m_{al} + m_b}$$

$$p_{al} = \mp\frac{3.31662\sqrt{m_{al}}\sqrt{m_b}}{\sqrt{m_{al} + m_b}} \qquad p_b = \pm\frac{3.31662\sqrt{m_{al}}\sqrt{m_b}}{\sqrt{m_{al} + m_b}}$$

We are interested in finding the energy of the alpha particle in this problem, and since we know the mass of the alpha particle and the daughter nucleus, the result is easily found. By substituting our known values of $m_{al} = 4$ and $m_b = 206$ into our derived E_{al}equation we get:

$$E_{al} = 5.395\ \text{MeV}$$

Note : We can obtain solutions for all the variables by substituting $m_b = 206$ and $m_{al} = 4$ into the derived equations above :

$$E_{al} = 5.395\ \text{MeV} \qquad E_b = 0.105\ \text{MeV} \qquad p_{al} = \mp 6.570\sqrt{\text{amu} * \text{MeV}} \qquad p_b = \pm 6.570\sqrt{\text{amu} * \text{MeV}}$$

▪ Problem 1.4. Calculation of Wavelength from Energy.

Since an x-ray must essentially be created by the de-excitation of a single electron, the maximum energy of an x-ray emitted in a tube operating at a potential of 195 kV must be 195 keV. Therefore, we can use the equation E=h∨, which is also E=hc/λ, or λ=hc/E. Plugging in our maximum energy value into this equation gives the minimum x-ray wavelength.

$$\lambda = \frac{h \times c}{E}$$ where we substitute $h = 6.626 \times 10^{-34}\ J * s$, $c = 299\,792\,458\ m/s$ and $E = 195$ keV

$$\lambda = \frac{\textbf{1.01869 J–m}}{\textbf{KeV}} = \textbf{0.0636 Angstroms}$$

Problem 1.5. ^{235}U Fission Energy Release.

Using the reaction $^{235}U \rightarrow {}^{117}\text{Sn} + {}^{118}\text{Sn}$, and mass values, we calculate the mass defect of:

$$M\,(^{235}U) - \left[M(^{117}\text{Sn}) + M(^{118}\text{Sn})\right] = \Delta\text{M}$$

and an expected energy release of $\Delta\text{M}c^2$.

$$Q = (235.0439 - (116.9029 + 117.9016))\ \text{AMU} \times \frac{931.5\ \text{MeV}}{\text{AMU}} = 223\ \text{MeV}$$

This is one of the most exothermic reactions available to us. This is one reason why, of course, nuclear power from uranium fission is so attractive.

Problem 1.6. Specific Activity of Tritium.

Here, we use the text equation Specific Activity = (ln(2)*Av)/ ($T_{1/2}$*M), where Av is Avogadro's number, $T_{1/2}$ is the half-life of the isotope, and M is the molecular weight of the sample.

$$\text{Specific Activity} = \frac{\ln(2) \times \text{Avogadro's Constant}}{T_{1/2}\,M}$$

We substitute $T_{1/2}$ = 12.26 years and M=$\frac{3\text{ grams}}{\text{mole}}$to get the specific activity in disintegrations/(gram–year).

$$\text{Specific Activity} = \frac{1.13492 \times 10^{22}}{\text{gram –year}}$$

The same result expressed in terms of kCi/g is shown below

$$\text{Specific Activity} = \frac{9.73\ \text{kCi}}{\text{gram}}$$

Problem 1.7. Accelerated particle energy.

The energy of a particle with charge q falling through a potential ΔV is qΔV. Since ΔV= 3 MV is our maximum potential difference, the maximum energy of an alpha particle here is q*(3 MV), where q is the charge of the alpha particle (+2). The maximum alpha particle energy expressed in MeV is thus:

$$\text{Energy} = 3\ \text{Mega Volts} \times 2\ \text{Electron Charges} = 6.\ \text{MeV}$$

- **Problem 1.8. Photofission of deuterium.** $^{2}_{1}D + \gamma \rightarrow {}^{1}_{0}n + {}^{1}_{1}p$ **+ Q (-2.226 MeV)**

The reaction of interest is $^{2}_{1}D + {}^{0}_{0}\gamma \rightarrow {}^{1}_{0}n + {}^{1}_{1}p$+ Q (-2.226 MeV). Thus, the γ must bring an energy of at least 2.226 MeV in order for this endothermic reaction to proceed. Interestingly, the opposite reaction will be exothermic, and one can expect to find 2.226 MeV gamma rays in the environment from stray neutrons being absorbed by hydrogen nuclei.

- **Problem 1.9. Neutron energy from D-T reaction by 150 keV deuterons.**

We write down the conservation of energy and momentum equations, and solve them for the desired energies by eliminating the momenta. In this solution, "a" represents the alpha particle, "n" represents the neutron, and "d" represents the deuteron (and, as before, "p" represents momentum, "E" represents energy, and "Q" represents the Q-value of the reaction).

$$p_a + p_n = p_d \qquad \frac{p_a^2}{2\,m_a} = E_a \qquad \frac{p_n^2}{2\,m_n} = E_n \qquad \frac{p_d^2}{2\,m_d} = E_d \qquad E_a + E_n = E_d + Q$$

Next we want to solve the above equations for the unknown energies by eliminating the momenta. (Note : Using computer software such as Mathematica is helpful for painlessly solving these equations).

We evaluate the solution by plugging in the values for particle masses (we use approximate values of "m_a," "m_n,"and "m_d" in AMU, which is okay because we are interested in obtaining an energy value at the end). We define all energies in units of MeV, namely the Q-value, and the given energy of the deuteron (both energy values are in MeV). So we substitute m_a = 4, m_n = 1, m_d = 2, Q = 17.6, E_d = 0.15 into our momenta independent equations. This yields two possible sets of solutions for the energies (in MeV). One corresponds to the neutron moving in the forward direction, which is of interest.

$$E_n = 13.340 \text{ MeV} \qquad E_a = 4.410 \text{ MeV}$$
$$E_n = 14.988 \text{ MeV} \qquad E_a = 2.762 \text{ MeV}$$

Next we solve for the momenta by eliminating the energies. When we substitute m_a = 4, m_n = 1, m_d = 2, Q = 17.6, E_d = 0.15 into these equations we get the following results.

$$p_n = \frac{p_d}{5} \mp \frac{1}{5}\sqrt{2}\,\sqrt{3\,p_d^2 + 352} \qquad p_a = \frac{1}{10}\left(8\,p_d \pm 2\sqrt{2}\,\sqrt{3\,p_d^2 + 352}\right)$$

We do know the initial momentum of the deuteron, however, since we know its energy. We can further evaluate our solutions for p_n and p_a by substituting:

$$p_d = \sqrt{2 \times 2 \times 0.15}$$

The particle momenta (in units of $\sqrt{\text{amu} * \text{MeV}}$) for each set of solutions is thus:

$$p_n = -5.165 \qquad p_a = 5.940$$
$$p_n = 5.475 \qquad p_a = -4.700$$

The largest neutron momentum occurs in the forward (+) direction, so the highest neutron energy of 14.98 MeV corresponds to this direction.

Radiation Interaction Problems

Problem 2.1 Stopping time in silicon and hydrogen.

Here, we apply Equation 2.3 from the text.

$$T_{stop} = \frac{1.2 \text{ range} \sqrt{\frac{\text{mass}}{\text{energy}}}}{10^7}$$

Now we evaluate our equation for an alpha particle stopped in silicon. We obtained the value for "range" from Figure 2.8 (converting from mass thickness to distance in meters by dividing by the density of Si $\simeq 2330 \text{ mg}/\text{cm}^3$). The value for "mass" is approximated as 4 AMU for the alpha particle, and the value for "energy" is 5 MeV. We substitute range = $22 \; x \; 10^{-6}$, mass = 4 and energy = 5 into Equation 2.3 to get the approximate alpha stopping time (in seconds) in silicon.

$$T_{stop} = 2.361 \times 10^{-12} \text{ seconds}$$

Now we do the same for the same alpha particle stopped in hydrogen gas. Again, we obtain the value for "range" (in meters) from Figure 2.8 in the same manner as before (density of H $\simeq$ .08988 mg/cm^3), and, of course, the values for "mass" and "energy" are the same as before (nothing about the alpha particle has changed). We substitute range = 0.1, mass = 4 and energy =5 into Equation 2.3 to get the approximate alpha stopping time (in seconds) in hydrogen gas.

$$T_{stop} = 1.073 \times 10^{-8} \text{ seconds}$$

The results from this problem tell us that the stopping times for alphas range from about picoseconds in solids to nanoseconds in a gas.

Problem 2.2. Partial energy lost in silicon for 5 MeV protons.

Clever technique: A 5 MeV proton has a range of 210 microns in silicon according to Figure 2-7. So, after passing through 100 microns, the proton has enough energy left to go another 110 microns. It takes about 3.1 MeV, according to the same figure, to go this 110 microns, so this must be the remaining energy. Thus the proton must have lost 1.9 MeV in the first 100 microns.

Problem 2.3. Energy loss of 1 MeV alpha in 5 microns Au.

From Figure 2.10, we find that $\frac{-1}{\rho} \frac{\text{dE}}{\text{dx}} \simeq 380 \; \frac{\text{MeV} * \text{cm}^2}{g}$. Therefore, $\frac{\text{dE}}{\text{dx}} \simeq 380 \; \frac{\text{MeV} * \text{cm}^2}{g} * \rho$ (ignoring the negative sign will not affect the result of this problem).

$$\text{Energy loss} = \frac{\rho \; (\text{dE}/\text{dx}) \; \Delta \text{x}}{\rho}$$

We substitute $\text{dE}/\text{dx} = \frac{380 \text{ MeV cm}^2 \rho}{\text{gram}}$, $\rho = \frac{19.32 \text{ grams}}{\text{cm}^3}$ and $\Delta \text{x} = 5$ microns to get the energy loss of the 1 MeV α-particle in 5 μm Au (in non-SI units).

$$\textbf{Energy loss} = \frac{\textbf{36 708 MeV microns}}{\textbf{cm}}$$

The result in SI units is thus:

$$\textbf{Energy loss} = 3.671 \times 10^{6}\ \textbf{eV}$$

Since this energy loss is greater than the initial energy of the particle, all of the α-particle energy is lost before 5 μm.
Note the small range of the α , i.e. ~ μm per MeV.

■ Problem 2.4. Range of 1 MeV electrons in Al. Scaling Law.

The Bragg-Kleeman rule, or scaling law, allows us to relate the known range in one material to the range in another material. The semi-empirical rule we use is:

$$\frac{R_1}{R_0} \cong \frac{\rho_0 \sqrt{A_1}}{\rho_1 \sqrt{A_0}} \text{ (Equation 2.7)}$$

Here, we use Figure 2.14 to approximate the 1 MeV electron range in silicon (R_0), and since we know every other quantity in Equation 2.7, we can approximate the range of the 1 MeV electron in aluminum (R_1). Solving for R_1 we can find the estimated range of the electron in Aluminum.

$$r_{\text{Al}} = \frac{r_{\text{Si}}\, \rho_{\text{Si}} \sqrt{\frac{\text{AW}_{\text{Al}}}{\text{AW}_{\text{Si}}}}}{\rho_{\text{Al}}}$$

We substitute $r_{\text{Si}} = \frac{0.5\,g}{\text{cm}^2\,\rho_{\text{Si}}}$, $\text{AW}_{\text{Al}} = 26.9815$ amu, $\text{AW}_{\text{Si}} = 28.0855$ amu, $\rho_{\text{Al}} = \frac{2.698\,g}{\text{cm}^3}$ and $\rho_{\text{Si}} = \frac{2.329\,g}{\text{cm}^3}$ to get the approximate range of 1 MeV electrons in aluminum (in cm).

$$r_{\text{Al}} = \textbf{0.1816 cm}$$

■ Problem 2.5. Compton scattering.

This problem asks for the energy of the scattered photon from a 1 MeV photon that scattered through 90 degrees. We use the Compton scattering formula (Equation 2.17). We write the Compton scattering formula, defining the scattering angle ("θ") as 90 degrees and the photon energy ("E_0") as 1 MeV.

$$\textbf{Energy} = \frac{E_0}{\frac{(1-\text{Cos}[\theta])\, E_0}{m_e * c^2} + 1}$$

We substitute $\theta = 90°$ and $E_0 = 1$ MeV to get the energy of the scattered photon in MeV.

$$\textbf{Energy} = \textbf{0.338 MeV}$$

Problem 2.6. Prob of photoelectric in Si versus Ge

For a rough estimate, we can note that photoelectric probabilities vary as $\sim Z^{4.5}$so we would expect that

$$\tau_{Si} / \tau_{Ge} = (14/32)^{4.5} = 0.0242$$

■ Problem 2.7. The dominant gamma ray interaction mechanism.

See Figure 2-20 and read off the answers (using the given gamma-ray energies and the Z-number for the given absorber in each part).
Compton scattering: a, b, and d.
Photoelectric absorption: c
Pair production: e

■ Problem 2.8. Mean free path in NaI of 1 MeV gamma-rays.

(a). The gamma-ray mean free path (λ) in NaI is $1/\mu$ (where μ is the total linear attenuation coefficient in NaI). The mass attenuation coefficient ($\frac{\mu}{\rho}$) is 0.06 cm^2/gm at 1 MeV according to Figure 2.18, and the density of NaI relative to water (ρ) is 3.67 gm/cm^3(by the definition of specific gravity). Therefore, we have $\lambda = 1/\mu = \frac{1}{\left(\frac{\mu}{\rho}\right) * \rho}$. Here, we will denote the mass attenuation coefficient ($\frac{\mu}{\rho}$) by μ_ρ, so we have

$$\lambda = \frac{1}{(\mu_\rho \, \rho)}$$

We substitute $\mu_p = \frac{0.06 \text{ cm}^2}{g}$ and $\rho = \frac{3.67 \, g}{\text{cm}^3}$to get the mean free path of 1 MeV gamma-rays in NaI (in cm).

$$\lambda = 4.54 \text{ cm}$$

(b). Any photon which emerges from 1 cm cannot have undergone a photoelectric absorption. Neglecting buildup factors, the probability that a photon emerges from the slab without having an interaction is $e^{-\mu_T x}$, where $\mu_T$is the **total** attenuation coefficient. The complement of this is the probability that a photon doesn't emerge from the slab without having had at least one interaction (1-$e^{-\mu_T x}$). The probability that the interaction is a photoelectric interaction is τ/μ_T(this is not the probability per unit path length, but the total probability that any given interaction is a photoelectric interaction). Therefore, the probability that a photon undergoes photoelectric absorption in the slab is (τ/μ_T)*(1-$e^{-\mu_T x}$). This equation is expressed below, along with the values for μ_T (which is just 1 divided by the previous result for λ), the attenuation distance (denoted "x" and which is 1 cm), and τ, which is just the mass attenuation coefficient for photoelectric absorption (found on Figure 2.18 to be 0.01) multiplied by the density of NaI (3.67 g/cm3).

$$\textbf{Probability of photoelectric absorption} = \frac{\tau \, (1 - e^{-\mu_T x})}{\mu}$$

We substitute $\mu_\tau = \frac{1}{4.54\text{ cm}}$, $x = 1$ cm and $\tau = \frac{0.01\times 3.67}{\text{cm}}$ to get the probability of photoelectric absorption for 600 keV gamma-rays in 1 cm NaI.

Probability of photoelectric absorption = 0.0329

What is interesting is that a different result is obtained using a different, although seemingly equally valid approach. We can note that the probability per unit path length of a photoelectric interaction is τ, so $1 - e^{-\tau x}$is the probability of a photoelectric interaction in traveling a distance x.

Probability of *a* photoelectric interaction = $1 - e^{-\tau x}$

We substitute x=1cm and $\tau = \frac{0.010\times 3.67}{\text{cm}}$to get the probability of a photoelectric interaction in traveling a distance x.

Probability of *a* photoelectric interaction = 0.0360

This result is slightly (10%) larger from the previous answer because this approach does not account for the attenuation of photons through the material by other means.

■ Problem 2.9. Definitions

See text.

■ Problem 2.10. Mass attenuation coefficient for compounds.

The linear attenuation coefficient is the probability per path length of an interaction. In a compound, the total linear attenuation coefficient would be given by the sum over the i^{th}constituents multiplied by the density of the compound:

$$\mu_c = \rho_c \sum w_i \left(\frac{\mu}{\rho}\right)_i \qquad \text{(Equation 2.23)}$$

where $w_i$is the weight fraction of the i^{th}constituent in the compound (represented by "w_H" and "w_O," respectively, in this problem), $(\mu/\rho)_i$ is the mass attenuation coefficient of the i^{th} constituent in the compound (represented by "μ_H" and "μ_O," respectively), and ρ_c is the density of the compound (represented by "ρ_W" for the density of water). The expression below shows this sum for water attenuating 140 keV gamma rays:

$$\boldsymbol{\mu_c = \rho_W\,(\mu_H\, w_H + \mu_O\, w_O)}$$

We substitute $\mu_H = .26\ \text{cm}^2/g$, $\mu_O = .14\ \text{cm}^2/g$, $\rho_W = 1\ g/\text{cm}^3$, $w_H = 2/18$ and $w_O = 16/18$ to get the linear attenuation coefficient for water attenuating 140 keV gamma rays.

$$\boldsymbol{\mu_c = \frac{0.153333}{\text{cm}}}$$

The mean free path, λ, is just the inverse of this last calculated value, or $\frac{1}{\mu_C}$. The mean free path of 140 keV gamma rays in water is thus:

λ_{H_2O} = 6.52 cm

This turns out to be an important result because Tc − 99 *m* is a radioisotope routinely used in medical diagnostics and it emits 140 keV gamma rays. Since most of the human body is made of water, this gives us an idea of how far these gamma rays can travel without an interaction through the human body and into our detectors.

▪ Problem 2.11. 1 J of energy from 5 MeV depositions.

We are looking for the number of 5 MeV alpha particles that would be required to deposit 1 J of energy, which is the same as looking for how many 5 MeV energy depositions equal 1 J of energy. We expect the number to be large since 1 J is a macroscopic unit of energy. To find this number, we simply take the ratio between 1 J and 5 MeV, noting that 1 MeV = 1.6 x 10^{-13} J.

The number of 5 MeV alpha particles required to deposit 1 J of energy is thus:

$$\boldsymbol{n} = \frac{\textbf{1 Joule}}{\textbf{5 MeV}} = \mathbf{1.25 \times 10^{12}} \textbf{ alpha particles}$$

▪ Problem 2.12. Beam energy deposition.

100 μA tells us the current, i.e., the number of Coulombs passing in unit time. If we divide this by the amount of charge per particle, it gives us the number of particles passing through the area in unit time. If we take this value, and multiply it by the energy per particle, we get the energy/time, or power. The power dissipated in the target by a beam of 1 MeV electrons with a current of 100 μA is thus:

$$\textbf{Power} = \frac{(\mathbf{100}\ \boldsymbol{\mu}\textbf{A})\ (\textbf{1 MeV})}{(\textbf{Electron Charge})} = \textbf{100 Watts}$$

This is a surprisingly low amount of power, about the same as from a light bulb, and represents the output from a small accelerating device.

▪ Problem 2.13. Exposure rate 5 m from 1 Ci of ^{60}Co.

We use the following equation for exposure rate:

Exposure Rate = $\Gamma_S \frac{\alpha}{d^2}$ (Equation 2.31)

where α is the source activity, d is the distance away from the source, and Γ_S is the exposure rate constant. The exposure rate constant for Co-60 is 13.2 $R - \text{cm}^2 / \text{hr} - \text{mCi}$ (from Table 2.1). The exposure rate 5 m from a 1 Ci Co-60 source is thus:

$$\textbf{Exposure Rate} = \Gamma_s \frac{\boldsymbol{\alpha}}{\boldsymbol{d}^2} \quad \text{where we substitute } \alpha = 1\text{ Ci},\ d = 5\,m,\ \Gamma_s = 13.2\ \frac{R - \text{cm}^2}{\text{hr} - \text{mCi}}$$

$$\textbf{Exposure Rate} = \frac{\mathbf{0.528\ cm^2}\ \boldsymbol{R}}{\mathbf{hr\ mm^2}} = \frac{\textbf{52.8 mR}}{\textbf{hr}}$$

The result in SI units :

$$\textbf{Exposure Rate} = \frac{\mathbf{3.78 \times 10^{-9}}\ \boldsymbol{C}}{\textbf{kg} - \boldsymbol{s}}$$

Problem 2.14. ΔT/Δt from 10 mrad/hr.

The dose of 1 rad corresponds to an energy deposition of 100 ergs/gram. So 10 mrad/hr corresponds to 1 erg/(gram-hr). Using a specific heat of water as 1 calorie/(gram $-^{o}$C), we can use the equation $\Delta Q = mC_p \, \Delta T$. Our given 1 ergs/(gram-hr) is $\Delta Q/(m*\Delta t)$. If we divide our equation by mC_p, we are left with ΔT/Δt on the right side, which is the quantity of interest, i.e.

$\Delta T/\Delta t = \dfrac{\Delta Q/(m\Delta t)}{C_p} = \dfrac{1 \text{ ergs/(gram}-\text{hr)}}{1 \text{ calorie/(gram } -^{o}C)}$.

$$\frac{\Delta T}{\Delta t} = \frac{1 \text{ Erg}}{\frac{1 \text{ Gram Calorie Hour}}{\text{Gram Centigrade}}}$$

The result below is the rate of temperature rise in a sample of liquid water with an absorbed dose rate of 10 mrad/h. Note that this is virtually impossible to measure because it is so small.

$$\frac{\Delta T}{\Delta t} = \frac{2.39 \times 10^{-8} \text{ Centigrade}}{\text{Hour}}$$

There are unusual radiation detectors which actually use the temperature rise in a detecting material to detect ionizing radiation. For the curious reader, do some research on bolometers in radiation detection, and also read about the superconducting radiation detectors under development.

■ Problem 2.15. Fluence-dose calculations for fast neutron source.

The Cf source emits fast neutrons with the spectrum N(E) dE given in the text by Eqn. 1.6. Each of those neutrons carries a dose h(E) that depends on its energy as shown in Fig. 2.22(b). To get the total dose, we have to integrate N(E)h(E) over the energy range.

First, let's consider only the neutron dose h(E). In the MeV range, we need a linear fit to the log h-log E plot of Figure 2.22(b) by using two values read off the curve:

$$\text{Log}_{10}\left(10^{-12}\right) = b + m \, \text{Log}_{10}(0.01)$$
$$\text{Log}_{10}\left(10^{-10}\right) = b + m \, \text{Log}_{10}(1.0)$$

Solving the system of equations above yields :

$$m = 1 \quad \text{and} \quad b = -10$$

This result gives us an approximate functional form for h(E) $\left[\text{in Sv} - \text{cm}^2\right] = 10^{-10}\, E$ [MeV]. Recall 1 Sv = 100 Rem = 10^5 mrem. In a moment, we'll integrate N(E)h(E) to get the total dose-area, but first check the normalization of N(E):

$$\text{norm} = \int_0^\infty \sqrt{E} \; e^{-\frac{E}{1.3}} \, dE \quad = \; 1.31359$$

We'll need this normalization factor because we will want to use N(E)/norm as the probability that a source neutron has energy E. Now, a source neutron – cm^2 arriving at the person delivers a dose (in Sv) of:

$$\text{neutron} - \text{dose} = \frac{\int_0^\infty E^{3/2} \, e^{-\frac{E}{1.3}} \, dE}{10^{10} \text{ norm}} \; = \; 1.95 \times 10^{-10} \text{ Sv}$$

We now need to multiply this by the number of neutrons produced by 3 micrograms of Cf-252 (2.3 x 10^6n/sec-mg) at 5 meters over 8 hours:

$$\textbf{dose} - \textbf{equivalent} = \frac{2.3 \times 10^6 \,(3\,\mu g)\,(8 \times 3600\ \text{sec})}{\mu\, g\ \ \text{sec}\left(4\,\pi\, 500^2\right)} \quad = \quad 63\,254.5\ \ \text{neutrons}\big/\text{cm}^2$$

So the total dose equivalent is given by (using 10^5mrem/Sv):

$$\textbf{dose equivalent}_{\text{neutrons}} = 1.23\ \text{mrem}$$

This is a small dose, comparable to natural background.

Aside: What about the dose from the gamma rays? They are high-energy gammas and the source emits 9.7 gammas per fission. Using a value of h_E~ 5 x 10^{-12} Sv – cm^2:

$$\textbf{dose equivalent}_{\gamma} = \frac{\left(2.3 \times 10^6\ \text{neutrons}\right)(3\,\mu g)\,(8 \times 3600\ \text{sec})\,(5\ \text{Sv cm}^2)\,(100\ \text{Rad})\,(1000\ \text{mRad})}{(1\,\mu g\ \ \text{sec})\left(10^{12}\ \text{neutrons}\right)(4\,\pi\, 500\ \text{cm}^2)\,(\text{Sv Rad})} = 0.0316\ \text{mRad}$$

So the gamma dose is even smaller than that from the fast neutrons, as expected. Fast neutrons have a high quality factor, i.e., they produce a heavy charged particle when they interact, and therefore do a lot more biological damage than the light electrons produced when gamma rays interact in materials.

Counting statistics problems

■ Problem 3.1. Effect of increasing number of trials on sample variance.

The relative variance of the variable x, i.e., $\frac{\sigma_x{}^2}{<x>} = \frac{<x^2>}{<x>^2} - 1$, is dependent only on the ratio of the means of x^2 and x. It does not depend upon the uncertainty in those quantities. Since these means are not expected to change with more samples, the relative variance (i.e., 2% of the mean) shouldn't change. Note that this conclusion is independent of the type of distribution (Poisson, Guassian, Binomial, etc.) for x. However, for any quantity that is derived from measurements, such as the mean <x>, the uncertainty in that quantity improves with additional measurements as shown by: $\sigma_{<x>} = \sqrt{\frac{<x>}{N}}$.

■ Problem 3.2. Probability of 8 heads occuring in 12 coin tosses.

We define the binomial distribution for n = 12 (12 trials), p = 0.5 (probability of a success is 1/2), and we give the value k = 8 (the number of successes we are interested in). We substitute the known values of n,p and k and evaluate the binomial distribution below to get the probability that exactly 8 heads (or tails for that matter) will occur in 12 tosses of a coin.

$$\textbf{Probability} = \frac{n!}{k!\,(n-k)!}\, p^k\,(1-p)^{n-k} = 0.121$$

■ Problem 3.3. Statistics of males occuring in random population samples.

The mean is well known to be (prob of success)*(number of trials) =0.75 N The probability of success of any one trial (drawing a male) is large (so Poisson statistics is not valid) and the sample size is only 15. Binomial statistics therefore apply. We substitute n = 15 and p = 0.75 in the equations below to find the mean ($\overline{x}$), variance (σ^2) and standard deviation (σ).

$$\overline{x} = n \times p = 11.25$$

$$\sigma^2 = n\,p\,(1-p) = 2.81$$

$$\sigma = \sqrt{\sigma^2} = \sqrt{n\,p\,(1-p)} = 1.68$$

■ Problem 3.4. Probability of no sixes in ten rolls of a dice.

Here, we define our probability distribution function as a binomial distribution with n=10 (number of trials), p=1/6 (probability of a success), and evaluating it at k=0 successes, which is the same as finding the probability that no sixes (or no fives, or fours, etc.) will turn up in ten rolls of the dice. The probability that no sixes will turn up in ten rolls of a dice is thus:

$$\textbf{Probability} = \frac{n!}{k!\,(n-k)!}\, p^k\,(1-p)^{n-k} = 0.162$$

Problem 3.5. Statistics of errors in computer program statements.

Poisson statistics applies to this problem because the probability of success (an error) is low, but the expected number of successes is not >~20 (the expected number of successes is 250/60 ≃ 4.17, so we cannot apply Gaussian statistics).

a) Here, we simply define the expected mean and standard deviation of a Poisson distribution with an expected number of successes ($\bar{x}$ = np) of 250/60. Of course, the expected number of successes is the same as the expected mean (they are both equal to np), and in Poisson statistics, the standard deviation is the same as the square root of the expected mean. This is reflected in the results shown below:

$$\bar{x} = \frac{250}{60} = 4.17$$

$$\sigma = \sqrt{\bar{x}} = 2.04$$

b) Next, we define a Poisson distribution with $\bar{x}$= 100/60 (expected number of successes), and evaluate the Poisson probability distribution function at k=0 successes. The probability that a 100-statement program will be free of errors is thus:

$$\text{Probability} = \frac{e^{-\bar{x}}\,\bar{x}^k}{k!} = 0.189$$

Problem 3.6. When is square-root of a number an estimate of uncertainty?

The only time the square root of a number is an estimate of its uncertainty is when the number is a direct sample of a population. For example, the number cannot have units associated with it (i.e. the square root of a value is not an estimate of its uncertainty when it is not a number of counts obtained by direct measurement).
(a). Yes. This is just a number of counts.
(b). Yes. This is just a number of counts.
(c). No. This is a processed number. One must use error propagation to determine the error in the quantity derived from the measurements.
(d). No. A rate has units and involves a division of the number of counts by time. In this case, error propagation says: $\sigma^2 = \frac{N}{T^2}$.
(e). No. An average is a processed quantity derived from the measured values. Error propagation says: $\sigma = \sqrt{x/N}$ where x is the expected value and N is the number of samples in the average.
(f). Yes. Although this is a processed quantity, it would yield the same result as if we had just counted for one 5 minute period. Using error propagation: σ = Sqrt ($N_1 + N_2$ + ... + N_5), so the error in the sum is just square root of the sum. This only works for sums, and not for subtractions.

Problem 3.7. Source + Background -> Net counts and uncertainty

We are asked to find net = (S+B) - B and σ_{net}. Finding $\sigma_{net}$is a straight forward error propagation since all counts are taken for 1 minute. Below we define the equation for the net counts and the standard error propagation formula (Eqn. 3.37). As a shorthand notation, the error propagation equation is expressed as a dot product between the two vectors representing the squared partial derivativies and the corresponding variances. The variable "sb" refers to (S+B) and "b" refers to B in the equation for "net".

$$\text{net} = \text{sb} - b$$

$$\sigma_{\text{net}} = \sqrt{\left\{\left(\frac{\partial\,\text{net}}{\partial\,\text{sb}}\right)^2, \left(\frac{\partial\,\text{net}}{\partial\,b}\right)^2\right\}.\{\sigma_{\text{sb}}{}^2, \sigma_b{}^2\}}$$

We substitute sb=561, b=410, $\sigma_{\text{sb}}{}^2 = \text{sb}$ and $\sigma_b{}^2 = b$ to get the net number of counts and the expected uncertainty in the net number of counts (σ_{net}).

net = 151 counts

σ_{net} **= 31.16**

■ Problem 3.8. Source + Background -> Net count rate and uncertainty

In this problem, the source plus background count rate (S+B) is 846 counts in 10 min, and the background count rate (B) is 73 counts in 10 min. We want the net count rate and its associated standard deviation. The net count rate is simply (S+B)-B, which is calculated below.

$$\text{net count rate} = \frac{846 - 73}{10} = \frac{77.3\ \text{counts}}{\text{minute}}$$

The expected error, or the standard deviation, in the count rates for (S+B) and B are calculated using the error propagation formula for division by a constant (Eq. 3.40), and then the standard deviation in the net count rate is calculated using error propagtion for differences of counts (Eq. 3.38), where σ_{S+B} and σ_B are substituted for σ_x and σ_y. The expected error, or standard deviation, in the net count rate (counts/min) is thus:

$$\sigma_{\text{net}} = \frac{\sqrt{846 + 73}}{10} = \frac{3.03\ \text{counts}}{\text{minute}}$$

■ Problem 3.9. Results using optimal counting times for Problem 3.8.

We first solve the equation giving the optimal division of time (Eq. 3.54). This is done below, where we have again defined the variables "sb" to denote the total counts of (S+B), and "b" to denote the background B in the equation. First we solve the system of equations for T_{sb} and T_b, and then we substitute the measured values for the count rates (84.6 and 7.3 counts/min, respectively) and the total amount of time allowed for the measurements to be done (20 min) to find the numerical values of $T_{\text{sb}}$and T_b.

$$T_{\text{sb}} = \sqrt{\frac{\text{sb}}{b}}\ T_b \qquad T_b + T_{\text{sb}} = T_{\text{tot}}$$

The resulting solutions for T_{sb} and T_b solved from the equations above is thus:.

$$T_{\text{sb}} = \frac{\sqrt{\frac{\text{sb}}{b}}\ T_{\text{tot}}}{\sqrt{\frac{\text{sb}}{b}} + 1} \qquad T_b = \frac{T_{\text{tot}}}{\sqrt{\frac{\text{sb}}{b}} + 1}$$

We substitute sb = 84.6, b = 7.3 and $T_{\text{tot}} = 20$ to get the numerical values of T_{sb} and T_b in minutes (i.e. the optimal times for measuring (S+B) and B, respectively).

T_{sb} **= 15.46 minutes** T_b **= 4.54 minutes**

We now calculate the uncertainty in the net count rate using error propagation. When defining the net count rate, we denote the number of counts over the new optimal time intervals as "n_{sb}" and "n_b," respectively, and the optimal counting times as "t_{sb}" and "t_b," respectively. We then use the basic formula for error propagation with the appropriate variables (as in problem 3.7, the dot between the two bracketed quantities signifies the dot product between the two, just as if we thought of them as two vectors). Next we substitute in known values to get the expected error in the net count rate.

$$\text{net} = \frac{n_{sb}}{t_{sb}} - \frac{n_b}{t_b}$$

$$\sigma_{\text{net count rate}} = \sqrt{\left\{\left(\frac{\partial\,\text{net}}{\partial n_{sb}}\right)^2, \left(\frac{\partial\,\text{net}}{\partial n_b}\right)^2\right\}.\left\{\sigma_{n_{sb}}{}^2, \sigma_{n_b}{}^2\right\}}$$

We substitute $\sigma_{n_{sb}}{}^2 = n_{sb} = 84.6\ t_{sb}$, $\sigma_{n_b}{}^2 = n_b = 7.3\ t_b$, $t_{sb} = 15.5$ and $t_b = 4.5$ to get the expected uncertainty in the net count rate when the optimal time intervals are used.

$\sigma_{\text{net count rate}}$ **= 2.66**

The improvement factor is thus 3.03/2.66.

■ Problem 3.10. Counting time versus uncertainty.

We are looking for the improvement in the relative uncertainty of a measurement by longer counting. We know:

$\frac{\sigma_N}{N} = \frac{\sqrt{N}}{N} = \frac{1}{\sqrt{N}} = \frac{1}{\sqrt{\text{RT}}}$, where N is the number of counts, R is the count rate, and T is the measurement time interval.

Since we have only one measurement, N is assumed to be our experimental mean (see Eq. 3.29). Since the count rate remains constant ($R_1 = R_0$, and $\frac{1}{R_1} = \frac{1}{R_0}$), we have :

$$\left[\left(\frac{\sigma_N}{N}\right)^2 T\right]_1 = \left[\left(\frac{\sigma_N}{N}\right)^2 T\right]_0 \qquad \text{or} \qquad T_1 = T_0 \left[\frac{\left(\frac{\sigma_N}{N}\right)_0}{\left(\frac{\sigma_N}{N}\right)_1}\right]^2$$

We already know that $\left[\frac{\left(\frac{\sigma_N}{N}\right)_0}{\left(\frac{\sigma_N}{N}\right)_1}\right] = \left(\frac{2.8}{1.0}\right)$, and that $T_0 = 10$ min. From here, calculating T_1 is very simple. The new counting time in minutes is thus:

counting time = $10\left(\frac{2.8}{1.0}\right)^2$ **= 78.4 minutes**

Since the original counting time was 10 minutes, 68.4 minutes must be added to reduce the statistical uncertainty from 2.8% to 1.0%.

Problem 3.11. Better to increase source or decrease background?

Recall our relationship that the relative uncertainty (fractional standard deviation squared, or ϵ^2) in the source is given by:

$$\frac{1}{T}\frac{\left(\sqrt{S+B}+\sqrt{B}\right)^2}{S^2} \quad \text{(from Eqn. 3.55)}$$

(a). If the S >> B, then this becomes $\frac{1}{\mathrm{ST}}$. Doubling the source strength is the best choice since the background is nearly irrelevant.

(b). If S << B, then this becomes $\frac{4\,B}{S^2\,T}$. Doubling source improves the ratio by 4 times, whereas halving background only improves ratio by 2 times. Doubling the source strength is again the best choice.

Problem 3.12. Probabilities of getting desired counts.

The expected number of counts in two minutes is (2 min)*(2.87 counts/min) = 5.74 counts. This is too small to apply Gaussian statistics, so we assume Poisson statistics to be accurate.

a) Here, we define a Poisson distribution with a mean of 5.74 (or 2.87*2) counts (since this is the only measurement made), and we use the Poisson probability density function (PDF):

$$\textbf{Probability} = \frac{e^{-\mu}\,\mu^k}{k!}$$

We substitute μ=5.74 and k=5 successes (i.e., counts) to get the probability that a given 2 minute count will contain exactly 5 counts.

Probability = 0.167

b) In order to determine the probability that at least one count will be recorded, we simply subtract the probability of 0 counts being recorded from 1 (since the integral of the probability distribution function from 0 to infinity is normalized to 1).

$$\textbf{Probability} = 1 - \frac{e^{-\mu}\,\mu^k}{k!}$$

We substitute μ=5.74 and k=0 to get the probability that a given 2 minute count will contain at least 1 count:

Probability = 0.997

For the last part, we want to know how many counts are needed to ensure at least one count with probability >99%. Note that this is the same as looking for the measurement time "t" required to acheive this number of counts (i.e. number of counts = (2.87 counts/min)*t). The key to this problem is to note that the probability of 0 counts must be less than 1% to satisfy this condition. We again define our Poisson Distribution, but this time with a mean value of 2.87*t. We define the probability of observing no counts to be 1%, and we solve for the resulting value for "t" that satisfies this condition:

$$\frac{e^{-\mu}\,\mu^k}{k!} = .01$$

We substitute μ=2.87t and k=0 then solve for t to get the minimum counting time required (in minutes) to ensure with > 99% probability that at least one count is recorded:

$t = 1.61$ **minutes**

■ Problem 3.13. Percent standard deviation between the activity ratio of two sources.

The ratio of the activity of Source B to Source A is simply given by:

$$\text{Ratio} = \frac{[(\text{Source } B + \text{background count rate}) - (\text{background count rate})]}{[(\text{Source } A + \text{background count rate}) - (\text{background count rate})]}$$

This is represented below where we denote the number of counts by "c," measurement time by "t," background by "b," Source B + background by "bb," and Source A + background by "ab."

$$\textbf{Ratio } R = \frac{\frac{c_{bb}}{t_{bb}} - \frac{c_b}{t_b}}{\frac{c_{ab}}{t_{ab}} - \frac{c_b}{t_b}}$$

We substitute $c_{ab} = 251$, $t_{ab} = 5$, $c_{bb} = 717$, $t_{bb} = 2$, $c_b = 51$ and $t_b = 10$ to get the ratio of the activity of Source B to Source A.

Ratio $R = 7.84$

Next, we define the explicit error propagation formula with the appropriate variables (as in previous problems, using the dot product notation under the square root) and give the known values in counts and minutes, respectively.

$$\sigma = \sqrt{\left\{\left(\frac{\partial R}{\partial c_{ab}}\right)^2, \left(\frac{\partial R}{\partial c_{bb}}\right)^2, \left(\frac{\partial R}{\partial c_b}\right)^2\right\}.\{c_{ab}, c_{bb}, c_b\}}$$

We substitute $c_{ab} = 251$, $t_{ab} = 5$, $c_{bb} = 717$, $t_{bb} = 2$, $c_b = 51$ and $t_b = 10$ to get the percent standard deviation in the ratio of the activity of Source B to Source A.

$\sigma_{B/A} = 0.635$

■ Problem 3.14. Estimating source measurement time based on desired error.

This problem is asking us to find the minimum measurement time interval necessary for the (source + background) counting rate measurement such that the fractional standard deviation of the net source counting rate (source alone) is at most 3%. To do this, we simply write the equation for the fractional standard deviation, set it equal to .03, and solve for the unknown time interval for the source + background.

Below, we define the net counting rate as "R," "c" denotes a number of counts, "t" denotes a measurement time interval, "sb" indicates a measurement for source + background, and "b" indicates a measurement for background. Next we define the fractional standard deviation for "R" (which is $\frac{\sigma_R}{R}$) using the explicit form of the error propagation formula in the numerator in dot product notation, as in previous problems. We then set $\frac{\sigma_R}{R}$ equal to .03, substitute in known values and solve for t_{ab}.

$$R = \frac{c_{sb}}{t_{sb}} - \frac{c_b}{t_b}$$

$$\frac{\sigma_R}{R} = \frac{\sqrt{\left\{\left(\frac{\partial R}{\partial c_{sb}}\right)^2, \left(\frac{\partial R}{\partial c_b}\right)^2\right\}.\{c_{sb}, c_b\}}}{R} = 0.03$$

We substitute $c_{sb} = 80\, t_{ab}$, $c_b = 845$ and $t_b = 30$ then solve the above equation to get the time interval the source should be counted for (with background) to determine the counting rate due to the source alone to within a fractional standard deviation of 3% (in minutes).

$$t_{ab} = 54.1 \text{ minutes}$$

■ Problem 3.15. Uncertainty in groups of measurements.

(a). The data fluctuations are expected to be statistical if the standard deviation of the sample population is the square root of the mean. This seems true, but we check this with a Chi-squared distribution to be sure.

Here, we define the student's data set and calculate the standard deviation of that data set.

data = {25, 35, 30, 23, 27}

$$\sigma = 4.69$$

Next, we take the square root of the mean (which is 28).

$$\sqrt{\bar{x}} = 5.29$$

Here, we define chi-squared (χ^2) for the data set (using Eqn. 3.36).

$$\chi^2 = \frac{(N-1)\,\sigma^2}{\bar{x}}$$

We substitute N=5, σ^2=22 and $\bar{x}$=28 to get the value of $\chi^2$for our data set.

$$\chi^2 = \frac{22}{7}$$

This is our "measured" value of χ^2 . One can also calculate the expected value of χ^2 from data values drawn from a predicted distribution. (The χ^2 distribution is defined as the distribution of the quantity $\sum_{i=1}^{n} x_i^2$, where the x_i are random variables following a normal distribution that has a unit variance and a mean value of zero).

To be consistent with the approach taken in the textbook, we actually want to calculate 1 minus the χ^2 "**C**umulative **D**istribution **F**unction" (CDF). The χ^2 CDF is the integral from zero up to some argument, which in this case is $\frac{22}{7}$. The function CDF(x) gives the probability that the expected value of χ^2 ranges between 0 and the value x, assuming the data follow the normal distribution. The complement of this is then the probability that the expected value of χ^2 is larger than this value, which is what the textbook uses. One can look these values up in statitics tables, but we use *Mathematica* to do this calculation for us:

$$1 - \text{CDF}\left[\chi^2\left(4 \text{ degrees} - \text{of} - \text{freedom}, \frac{22}{7}\right)\right] = 0.534$$

Because this value is close to 0.5, the data are random (i.e. a true Poisson distribution would have a χ^2 probability of 0.5). As a side note, this probability could also have been estimated using the χ^2 distribution table.

b) This question is asking that since the data appears to be random, what is the EXPECTED standard deviation of the MEAN of 5 single measurements using just these data. For this data set, given a mean , $\sigma_{\bar{x}} = \sqrt{\frac{\bar{x}}{N}}$ (Eqn. 3.44).

Using the provided data, the expected standard deviation in the mean of the data set is:

$$\boldsymbol{\sigma_x = \sqrt{\frac{\bar{x}}{N}} = 2.37}$$

(c). Now, suppose we have the 30 measurements of the mean from the 30 students. What would we expect to measure for the variance of the set of these mean values $\{x_1, x_2, x_3,.., x_{30}\}$?

The variance estimated in (b) above IS the expected fluctuation when samples are drawn identically from that population. So we expect that the 30 mean values WILL show this variance (i.e. we would expect the sample variance in this situation to be the result above squared, or $s^2 = \sigma^2_{\bar{\bar{x}}}$; this is calculated to be ≃ 5.59).

(d). Suppose we now average these 30 values to get a better estimate of the mean. What is the standard deviation for the mean?

One way to look at this is to see it as 5 x 30 data points and calculate $\sigma_{\bar{\bar{x}}}$. The standard deviation goes down by $\sqrt{30}$, so the expected standard deviation of the mean when we use all 30 mean values will be 0.432049.

Another way is to calculate the standard deviation of the average of the averages. If we define $<\bar{x}>$ as the average of the 30 students individual means, then:

$$\sigma_{<\bar{x}>} = \sqrt{\frac{<\bar{x}>}{N}}$$

which, of course, turns out to be exactly the same formula.

■ Problem 3.16. Chi-squared test on a data set of counting measurements.

Define the data set we are applying the chi-squared (χ^2) test to as the variable "data":

data = {3626, 3731, 3572, 3689, 3625, 3711, 3617, 3572, 3578, 3569, 3677, 3630, 3615, 3605, 3591, 3678, 3624, 3652, 3595, 3636, 3465, 3574, 3601, 3540, 3629}

We determine the χ^2 value of the data by using the data values to calculate the sample variance (s^2) and the sample mean $\bar{x}$, and then calculate:

$$\chi^2_{\text{data}} = \frac{(\text{No. of data values} - 1)\ \text{Variance}(\text{data})}{\text{Mean}(\text{data})}$$

Using a calculator to determine the mean and variance of the data set, one finds the mean is 3616.08 and χ^2 is 21.2191.

In problem 3.15(a) above, we described how to determine the probability that a true Poisson distribution would have fluctuations larger than the data set by using the complement to the cumulative probability function of the χ^2 distribution.

We used *Mathematica* to calculate this probability, but it can also be found in your textbook or in standard tables of probabilities.

1 - CDF[ChiSquareDistribution(No. of data points - 1, χ^2_{data})] = 0.626

Since this $\chi 2$ probability is close to 0.5, the fluctuations are consistent with random fluctuations (relatively close to a true Poisson distribution). In general, one looks for values that are very small (~5%) or very large (~95%) to indicate that the data is not following the expected statistical model. If this is the case, then one looks at whether there is a problem with the measurement system.

■ Problem 3.17. Uncertainty in the difference between two measurements.

Suppose we are given that a set of I counts $\{N_i\}$, each taken over a period of time $t_I$results in an average rate $<r>$. The uncertainty in the average rate can be determined from error propagation once we write the average rate in terms of the measured counts:

$<r> = (1/I)(N_1/t_I + N_2/t_I + ... N_I/t_I) = (1/It_I)\,(N_1 + N_2 + ... N_I) = N_{total}/t_{total}$
so
$\sigma_{<r>}{}^2 = (1/It_I)^2\,\{N_1 + N_2 + \;...\; + \;N_I) = \;N_{total}\big/t_{total}{}^2$

We know that the total number of counts $N_{total} = \;<r>\; t_{total}$. Since for this problem, we are given <r> and $t_{total}$for each group, we can find $N_{total}$for each group.

Wtih N_A and N_B (total counts from Group A and Group B), we want to find out if the difference between $< r >_A$and $< r >_B$ is significant. Define this difference as:
$\Delta = \frac{N_A}{T_A} - \frac{N_B}{T_B}$.
where $T_A$is the total time for Group A (i.e., I * t_I), and similarly for Group B.

Using error propagation on these independent measurements,
$\sigma_\Delta{}^2 = \left(\frac{\sigma_{N_A}}{T_A}\right)^2 + \left(\frac{\sigma_B}{T_B}\right)^2 = (<r_A>/T_A + <r_B>/T_B)$, since $\sigma_{N_A}{}^2 = N_A$ (i.e., the standard deviation in a number of counts is equal to the square root of that number).

If both groups are making identical measurements, we expect the probability of observing a given value of Δ, i.e., P(Δ), to be Gaussian with <Δ>=0 and $\sigma_\Delta = \sqrt{<r>\left(\frac{1}{T_A} + \frac{1}{T_B}\right)}$

We look at our measured value of Δ, and see if it lies within $\pm\sigma_\Delta$ of 0. We will look at $\frac{\Delta_{meas}}{\sigma_{\Delta_{theory}}}$and if this value is >> 1, then the difference is significant since the probability of observing our value of Δ would be small.

Below we define $\frac{\Delta}{\sigma_\Delta}$, and substitute the known values for "N_A," "N_B," "T_A," and "T_B"

$$\frac{\Delta}{\sigma_\Delta} = \frac{\frac{N_A}{T_A} - \frac{N_B}{T_B}}{\sqrt{\frac{N_A}{T_A^2} + \frac{N_B}{T_B^2}}}$$

We substitute $N_A = 2162.4 \times 10,\; N_B = 2081.5 \times 20,\; T_A = 10$ and $T_B = 20$ to get the value of $\frac{\Delta}{\sigma_\Delta}$.

$$\frac{\Delta}{\sigma_\Delta} = 4.52$$

Since the difference between the two measured averages is 4.5 standard deviations from the expected mean (of zero), the difference is significant (i.e. there is a very small probability of observing such a value). We conclude that the premise that both groups were making the same measurement is highly unlikely.