MEASUREMENT AND REPRESENTATION OF SENSATIONS

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Edited by

HANS COLONIUS

EHTIBAR N. DZHAFAROV

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AND
REPRESENTATION
OF
SENSATIONS

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Foreword

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An important open issue in many areas of mathematical behavioral science concerns the extent to which probabilistic (nondeterministic) models are necessary to explain the data. This issue is distinct from, though related to, statistical issues that arise in testing deterministic models. To a significant extent, when researchers do propose probabilistic interpretations of the data, they leave the source of the underlying variability unspecified – this is less so in the study of psychophysics than in, say, the study of choice or voting behavior. As this book shows, both the deterministic and the probabilistic perspective are contributing significantly to modern psychophysics.

The book, which should interest both behavioral scientists and applied mathematicians, includes a sample of the most sophisticated current mathematical approaches to psychophysical problems. Most of the problems studied are classical, dating back to Fechner, von Helmholtz, Schrödinger, Stevens, and other founders of modern psychophysics. However, the techniques – both deterministic and probabilistic – presented in the book's six chapters are all original and recent. The chapters present rigorous mathematical definitions of theoretical concepts and discuss relatively simple procedures for the empirical evaluation of these concepts.

The volume, although not comprehensive in its coverage, encompasses a broad spectrum of psychophysical problems and approaches. Dzhafarov and Colonius, and, separately, Zhang, discuss probabilistic models of (subjective) similarity. In their first chapter, Dzhafarov and Colonius show that if probabilistic same-different judgments satisfy two quite general properties, which appear to hold for available data, then a large class of probabilistic models for such judgments are ruled out; in the second chapter, they apply one of these principles in a novel manner to derive subjective metrics from probabilistic same-different judgments. Zhang's chapter presents a somewhat similar approach and applies a variant of one of Dzhafarov and Colonius's general principles to situations where the two stimuli being compared have qualitatively distinct psychological representations. Luce and Steingrimsson present behavioral conditions that are sufficient for a deterministic representation of the psychophysical and weighting function involved in magnitude production. These behavioral conditions are formulated in terms of the joint effect of pairs of stimuli and of judgments of intervals separating two pairs of stimuli. Townsend, Aisbett, Busemeyer, and Assadi define and classify the possible forms of perceptual separability.

They do so by combining the language of differential geometry with general recognition theory, the latter being a multidimensional generalization of signal detection theory proposed earlier by Ashby and Townsend. And Balakhrishnan defines carefully the concept of observable probabilities and illustrates their use in the evaluation of the (sub)optimality of a decision rule. In doing so, he proposes a new probabilistic language that is applicable to all psychophysical tasks in which a participant's responses can be classified as either correct or incorrect, and uses this language to show how the classical concepts of psychophysical decision making (such as in the theory of signal detection) can be defined directly in terms of observable properties of behavior.

After thinking about this book and the possible strengths and weaknesses of the presented modeling approaches, I concluded that several factors encourage researchers to focus their attention on either deterministic or probabilistic models, usually to the relative exclusion of the other model class. The factor that I want to consider here is the complexity of the empirical situation. Focussing on psychophysics, there are at least two places where the empirical situation can be less or more complex: first, in the physical complexity of the stimuli; second, in the complexity of the task posed to the participant. I consider each in turn.

First, consider the complexity of the stimuli. For instance, consider a task where the participant is asked to make a "same-different" judgment, such as is studied in this book by Dzhafarov and Colonius, and by Zhang. This might be considered a fairly "simple" judgment to make. Now consider possible stimulus spaces for such a "same-different" task. If lines of varying length are the stimuli, then we have a relatively "simple" stimulus space, whereas if the stimuli are small spatiotemporal patches differing in color[1], then we have a "complex" stimulus space. There are then two relatively standard ways to carry out the experiment. In one, various pairs of stimuli are presented and the participant has to decide whether they are the "same" or "different"; in the second, one stimulus is presented and the participant has to adjust a second one until it "matches" the first. Independent of the experimental task, if the behavior is deterministic, then I think that, in the line length case, we will be surprised if there is more than one stimulus that "exactly" matches another[2]. However, in the color case,

[1] Note that line length can be measured physically, whereas "color" depends on the visual system being studied. However, in both cases, the stimuli being used can be specified in terms of physical variables. Also, when participants are asked to make color judgments, they are instructed to ignore other qualities of the stimulus such as hue or saturation.

[2] The matching lines may not be of the same length due to biases such as time-order effects. However, I think such effects can be considered minor in terms of the points I wish to make.

there will be a subspace of the stimulus space that matches any given color. Thus, assuming deterministic data, there is relatively little information in the "same-different" line length judgments, whereas there is considerable information in the "same-different" color judgments. This perspective is confirmed by the extensive deterministic representational theory concerning (metameric) color matching and its empirical evaluations, with no parallel (deterministic) theory and data concerning the matching of line-lengths. Of course, the data are probabilistic – or at least "noisy" – in both the line length and the color task when the stimuli are (psychologically) very similar. Thus, one would expect the development of probabilistic models for such "local" judgments, and attempts to use these "local" models and data to develop "global" representations. This is the approach taken in this book by Dzhafarov and Colonius, and by Zhang.

Second, consider the complexity of the participant's task. For instance, as in the chapter by Luce and Steingrimsson, assume that the basic stimulus is a pair (x, u) where x is a pure tone of some fixed frequency and intensity presented to the left ear of a participant and u is a pure tone of the same frequency and phase but a (possibly) different intensity presented to the right ear. The basic task is to judge which of two such pairs, (x, u) and (y, v), is the louder. However, in line with the intent of this paragraph to consider more complex tasks, now consider the additional task of *ratio production*, which involves the presentation to a participant of a positive number p and the stimuli (x, x) and (y, y), with y less intense than x, and asking the participant to produce the stimulus (z, z) for which the loudness "interval " from (y, y) to (z, z) is perceived to stand in the ratio p to the loudness "interval " from (y, y) to (x, x). As mentioned above, Luce and Steingrimsson show that, under a specific set of deterministic behavioral conditions, there is a numerical representation of these judgments that involves a psychophysical and a weighting function. And, though their data are somewhat "noisy" (probabilistic?), the behavioral properties are quite well-supported by the data.

Summarizing the above ideas, it appears that one can develop, and test, interesting deterministic psychological representations both for complex stimuli in simple tasks and for simple stimuli in (relatively) complex tasks. Of course, one can then combine these approaches to study complex stimuli in complex tasks. This is not to deny that there is likely some nondeterminism in each set of data, and that locally – when the stimuli are quite (psychologically) "similar" – there may be considerable nondeterminsm. Thus, a major challenge is to develop and test theories that have "local" nondeterminism in conjunction with "global" determinism. The chapters in this book present significant contributions to various parts of this challenge, some emphasizing "local" nondeterminism, some emphasizing “global” determinism, and some dealing with both aspects of the problem. I look forward to future work that builds on these sophisticated results by further integrating the study of “local” (nondeterminsitic) and “global” (deterministic) representations, continues the authors’ initial contributions to the study of dynamic effects, such as sequential dependencies, and extends the approaches to include response times.

1
Regular Minimality: A Fundamental Law of Discrimination

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1. INTRODUCTION

The term *discrimination* in this chapter is understood in the meaning of *telling stimuli apart*. More specifically, it refers to a process or ability by which a perceiver judges two stimuli to be different or identifies them as being the same (overall or in a specified respect). We postpone until later the discussion of the variety of meanings in which one can understand the terms *stimuli*, *perceiver*, and *same–different judgments*. For now, we can think of discrimination as pertaining to the classical psychophysical paradigm in which stimuli are being chosen from a certain set (say, of colors, auditory tones, or geometric shapes) two at a time, and presented to an observer or a group of observers who respond by saying that the two stimuli are the same, or that they are different. The response to any given pair of stimuli $(\mathbf{x}, \mathbf{y})$ in such a paradigm can be viewed as a binary random variable whose values (same–different) vary, in the case of a single observer, across the potential infinity of replications of this pair, or, in the case of a group, across the population of observers the group represents. As a result, each stimulus pair $(\mathbf{x}, \mathbf{y})$ can be assigned a certain probability, $\psi(\mathbf{x}, \mathbf{y})$, with which a randomly chosen response to $\mathbf{x}$ and $\mathbf{y}$ (paired in this order) is "the two stimuli are different,"

$$\psi(\mathbf{x}, \mathbf{y}) = \Pr[\mathbf{x} \text{ and } \mathbf{y} \text{ are judged to be different}]. \tag{1}$$

The empirical basis for considering $(\mathbf{x}, \mathbf{y})$ as an ordered pair, distinct from $(\mathbf{y}, \mathbf{x})$, is the same as for considering $(\mathbf{x}, \mathbf{x})$ as a pair of two identical stimuli rather than a single stimulus. Stimuli $\mathbf{x}$ and $\mathbf{y}$ presented to a perceiver for comparison are necessarily different in some respect, even when one refers to them as being physically identical and writes $\mathbf{x} = \mathbf{y}$: thus, $\mathbf{x}$ (say, a tone) may be presented first and followed by $\mathbf{y}$ (another tone, perhaps otherwise identical to $\mathbf{x}$); or $\mathbf{x}$ and $\mathbf{y}$ (say, aperture colors) may be presented side-by-side, one on the left, the other on the right. Dzhafarov (2002b) introduced the term *observation area* to reflect and generalize this distinction: two stimuli being compared belong to two distinct observations areas (in the examples just given, spatial locations, or ordinal positions in time). This seemingly trivial fact plays a surprisingly prominent role in the theory of perceptual discrimination. In particular, it underlies the formulation of the law of Regular Minimality, on which we focus in this chapter.

There is more to the notion of an observation area than the difference between spatiotemporal locations of stimuli, but this need not be discussed now. Formally, we refer to $\mathbf{x}$ in $(\mathbf{x}, \mathbf{y})$ as belonging to the *first observation area*, and to $\mathbf{y}$ as belonging to the *second observation area*, the adjectives "first" and "second" designating the ordinal positions of the symbols in the pair rather than the chronological order of their presentation. The difference between the two observation areas, whatever their physical meaning, is always perceptually conspicuous, and the observer is supposed to ignore it: thus, when asked to determine whether the stimulus on the left (or presented first) is identical to the stimulus on the right (presented second), the observer would normally perceive two stimuli rather than a single one, and understand that the judgment must not take into account the difference between the two spatial (or temporal) positions. In the history of psychophysics, this aspect of discrimination has not received due attention, although G. T. Fechner did emphasize its importance in his insightful discussion of the "non-removable spatiotemporal non-coincidence" of two stimuli under comparison (1887, p. 217; see also the translation in Scheerer, 1987).

It should be noted that the meaning of the term *discrimination*, as used by Fechner and by most psychophysicists after him, was different from ours. In this traditional usage, the notion of discrimination is confined to semantically unidimensional attributes (such as loudness, brightness, or attractiveness) along which two stimuli, $\mathbf{x}$ and $\mathbf{y}$, are compared in terms of which of them contains more of this attribute (greater–less judgments, as opposed to same–different ones). Denoting this semantically unidimensional attribute by $\mathcal{P}$, each ordered pair $(\mathbf{x}, \mathbf{y})$ in this paradigm is assigned probability $\gamma(\mathbf{x}, \mathbf{y})$, defined as

$$\gamma(\mathbf{x}, \mathbf{y}) = \Pr[\mathbf{y} \text{ is judged to be greater than } \mathbf{x} \text{ with respect to } \mathcal{P}]. \quad (2)$$

As a rule, although not necessarily, subjective attribute $\mathcal{P}$ is being related to its "physical correlate," a physical property representable by an axis of nonnegative reals (e.g., sound pressure, in relation to loudness). In this case, stimuli $\mathbf{x}, \mathbf{y}$ can be identified by values x, y of this physical property, and probability $\gamma(\mathbf{x}, \mathbf{y})$ can be written as $\gamma(x, y)$.[3] The physical correlate is always chosen so that $y \rightarrow \gamma(x, y)$; (i.e., function γ considered as a function of y only, for a fixed value of x) is a strictly increasing function for any value of x, as illustrated in Fig. 1, left. Clearly then, $x \rightarrow \gamma(x, y)$ is a strictly decreasing function for any value of y. Note, in Fig. 1 (left), the important notion of a *Point of Subjective Equality* (PSE). The difference between x, in the first observation area, and its PSE in the second observation area, is sometimes called the *constant error* associated with x (the term "systematic error" being preferable, because the difference between x and its PSE need not be constant in value across different values of x). The systematic error associated with y, in the second observation area, is defined analogously.

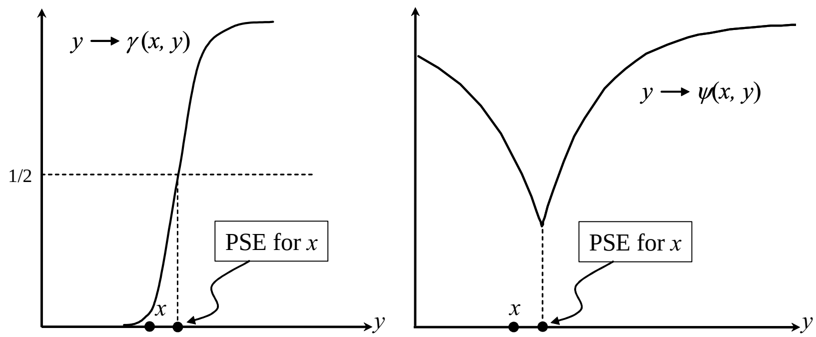

Fig. 1: Possible appearance of discrimination probability functions $\gamma(x, y) = \Pr[y \text{ is greater than } x \text{ in attribute } \mathcal{P}]$ (left) and $\psi(x, y) = \Pr[x \text{ is different from } y]$ (right), both shown for a fixed value of x, with x and y represented by real numbers (unidimensional stimuli). For $\gamma(x, y)$, the median value of y is taken as the Point of Subjective Equality (PSE) for x (with respect to $\mathcal{P}$). For $\psi(x, y)$, PSE for x is the value of y at which $\psi(x, y)$ achieves its minimum.

Same–different discrimination also may involve a semantically unidimensional attribute (e.g., "do these two tones differ in loudness?"), but it does not have to: the question can always be formulated "generically": are the two stimuli different (in anything at all, ignoring however the difference

[3] Here and throughout, we use boldface lowercase letters to denote stimuli, and lightface lowercase letters when dealing with their real-number attributes; by convenient abuse of language, however, we may refer to "stimulus x" in place of "stimulus $\mathbf{x}$ with value x."